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SOLUTION OF SYSTEMS OF LINEAR EQUATIONS ON L. I. GUTENMAKER'S ELECTRICAL MODELS

[Digest]

I. S. Gradshteyn

[Note: The above is the title of an article written by I. S. Gradshteyn and appearing in the Izvestiya Akademii Nauk SSSR, Otdeleniye Tekhnicheskikh Nauk, No. 5 (1947), pages 529-584. The article was read 11 Nov 1946 at the seminar on precision mechanics and computational techniques, given by the Section of Precision Mechanics in the Machine Building Institute, Academy of Sciences. This article is summarized below.]

The present work is devoted to the problem of stability in the solving of systems of ordinary linear differential and algebraic equations expressed in the matrical scheme proposed by L. I. Gutenmakher ("Artificial Reproduction of Physical Phenomena for the Solution of Technical Problems", in IAN, OTN Nos 4, 5, 1945; "Electrical Circuits for the Approximate Solution of Systems of Equations", Doklady Akademi Nauk SSSR, Vol 47, No. 5 (1945); and "Electrical Schemes for the Solutions of Systems of Equations", Elektrichestvo, No. 4 (1946), co-authors N. V. Korol'kov and V. A. Taft). This work is concerned with finding the cases where the scheme operates stably; that is, does not generate. The results obtained can be formulated thus:

Let there be given systems of linear differential equations with constant coefficients:

$$\sum_{j=1}^{n} (a_{ij} + b_{ij} p) u^{(j)} = \varphi_i(t) \qquad (i = 1, 2, \ldots, n).$$

The stability of operation of a matrical scheme for the solution of this system depends only upon the coefficients of the derivatives and not upon the coefficients of the unknowns. The scheme will operate stably if the

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coefficients b_{ii} of the derivatives standing in the main diagonal are positive and the matrix of coefficients, $\|b_{ij}\|$, is triangular. The triangularity of matrix $\|b_{ij}\|$ is merely a sufficient condition for stable operation of the scheme; the positiveness of the diagonal coefficients b_{ii} in the triangular matrix is a necessary condition.

If the matrix $\|b_{ij}\|$ of coefficients of the derivatives are degenerate, the scheme will generally speaking generate. If, however, in the matrix of coefficients of the derivatives any line consists of complex zeroes, then the stability of operation of the scheme depends upon that matrix in which the coefficients of the sought-for functions are set in the corresponding line. Thus, if the first of our equations is algebraic - that is, of the form

$$\sum_{j=1}^{n} a_{1j} u^{(j)} = \psi_1$$

then the stability depends upon the following matrix:

$$\left\| \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{matrix} \right\|$$

If all the given equations are algebraic, then the stability depends upon the matrix $\|a_{ij}\|$, as naturally expected.

For stability of operation of matricol schemes with amplifiers during the solution of algebraic equations, it is sufficient for this system that the sequential iterative process be convergent. ~~Necessary to~~ the "limit" criterion of convergence of this iteration is necessary. A wide class of systems of equations is easily transformed so that they could be solved in the matrix scheme. In particular, all those equations will be solved in a matrix scheme which satisfy any of the sufficient criteria of convergence of an

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iterative process, as indicated in R. Mises' article "Verfahren der Gleichungauflosung", ZAMM, Vol 9, pp 62-68, 1929. (Note: In this work, the sufficient criterion of convergence of the iterative process according to Seidel will be used).

All that has been said concerning the stability of solutions of systems of algebraic equations applies also to systems of differential equations. In this latter case all considerations concerning the convergence of the iterative process must refer to the matrix of coefficients of the derivatives. Therefore, during a solution of systems of linear differential equations, it is sufficient in most cases to regulate the system in a suitable manner ~~the system~~ in order that ~~the system~~ it might not generate.

This article contains certain results of two works published by the author and V. A. Taft - ("Electrical Modeling of Physical Processes with the Aid of Matrical Schemes with Amplifiers", in IAN, OTN Nos 1 and 3, (1946). In particular, the operator coefficient of amplification of a one-stage amplifier was evolved by V. A. Taft and later improved by N. I. Shteyn. N. I. Shteyn also gave the differential equations that describe the operation of a one-stage amplifier with inverter stage.

Formulas for setting up the scheme of initial conditions were proposed by N. V. Korob'kov, who gave a derivation for these formulas which differs from that given here. The theory, constructed by the authors, of the stability of operation of matrical schemes also permits one to investigate the error arising during solution of the equations.

The presentation is divided into two sections: A. Solving the Differential Equations, and B. Solving the Algebraic Equations, which in turn are divided into the following topics:

A:

1. Describing the Scheme for the Solution of a System of Linear Differential Equations with Constant Coefficients.
2. One-stage Amplifier; Differential Equations that Describe its Operation.

3. Stability of Operation of Matrical Schemes; Setting up the Problem.
4. Main Theorem on Stability.
5. One-stage Amplifier with Inverter Stage.
6. Operator Coefficient of Amplification.
7. Physical Significance of the Main Theorem on Stability.
8. Technical Execution of Stability of Operation of Schemes.
9. Assigning the Initial Values of the Desired Functions in a Matrical Scheme.

B.

1. Matrical Schemes by Resistances; Setting up the Problem on Stability.
2. Second Theorem on Stability.
3. Operation Coefficient of Amplification, and the Stability Criterion Governing the Stable Operation of a Scheme.
4. Solving a System of Linear Algebraic Equations by the Iteration Method.
5. Stability of Operation of a Matrical Scheme with Amplifiers, and the Convergence of the Iteration Process.
6. Stability of Operation of One Amplifier with Feedback.
7. Investigating the General Case: The Signs of ε_{ij} are Arbitrary.
8. Physical Significance of the Criterion of Stability (System of Two Equations).
9. Solving a System of Algebraic Equations in a Matrical Scheme with Capacitors.

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